第六屆培正數學邀請賽

6th Pui Ching Invitational Mathematics Competition

初賽（中一組）

Heat Event (Secondary 1)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

   There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

   All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

   The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

   The diagrams in this paper are not necessarily drawn to scale.

1. 當某數增大 120 時，它增大了 3 倍。求某數。 （3 分）

When a number is increased by 120, it is increased by 3 times. Find the number. (3 marks)

2. 某三角形的三隻內角分別是 27°、a° 和 b°，其中 a 和 b 是正整數。求 a 的最大可能值。 （3 分）

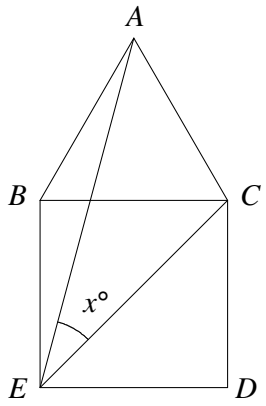
A triangle has interior angles 27°, a° and b°, where a, b are positive integers. Find the greatest possible value of a. (3 marks)

3. 小麗寫下了一個質數 A，而小婷則寫下了一個質數 B。已知 $A+B$ 是質數。若 $A<B$，求 A。 （3 分）

Alice wrote down a prime number A while Betty wrote down a prime number B. It is known that $A+B$ is prime. If $A<B$, find A. (3 marks)

4. 圖中，ABC 是等邊三角形，而 $BCDE$ 是正方形。若 $\angle AEC = x°$，求 x。 （4 分）

In the figure, ABC is an equilateral triangle while $BCDE$ is a square. If $\angle AEC = x°$, find x. (4 marks)



5. 小莉到百貨公司購物。她買了 10 件貨品，每件的價格（以「元」爲單位）都是整數。小莉爲了估算貨品的總值，把每件貨品的價格四捨五入至最接近 10 元然後加起來，並得到 1230 元的估算值。問貨品的確實總值最高是多少元？ （4 分）

Lily went shopping in a department store. She bought 10 items, and the price (in dollars) of each of them is an integer. To estimate the total price of the items, Lily rounded off the price of each item to the nearest $10 and added them up to get the estimated sum of $1230. What is the maximum possible value (in dollars) of the actual sum? (4 marks)

6. 當一個桶注水至三分之一滿時，其總重量是 600 克；當它注水至四分之三滿時，其總重量是 1200 克。當它注滿水時，其總重量是多少克？ （4 分）

When a tank is one-third-filled with water, its total weight is 600 grams. When it is three-fourths-filled with water, its total weight is 1200 grams. When the tank is full of water, what is its total weight (in grams)? (4 marks)

7. 某次數學競賽限時 1 小時 15 分鐘。一名參賽者發現，比賽期間他的手錶曾經 n 次出現時針和分針互相垂直。求 n 的最大可能值。 （4 分）

The time allowed for a mathematical competition is 1 hour 15 minutes. A contestant found that, during the contest, the minute hand and hour hand on his watch were perpendicular to each other at n instances. Find the greatest possible value of n. (4 marks)

8. 有多少個兩位正整數的數字之和是平方數？ （4 分）

How many two-digit positive integers have the property that the sum of its digits is a square number? (4 marks)

9. 從一副 52 張的撲克牌中，最少要抽出多少隻，才能確保當中有兩隻點數相同的紅色牌？ （5 分）

At least how many cards must be drawn from a pack of 52 to ensure that the cards drawn must contain two red cards of the same value? (5 marks)

10. 現有 5 塊邊長爲 1 的正方形咭片和 5 塊邊長爲 1 的等邊三角形咭片。若把這 10 塊咭片拼成一個多邊形，咭片之間不許互相重疊，那麼拼出的多邊形最少有幾條邊？ （5 分）

There are 5 square cardboards of side length 1 and 5 cardboards in the shape of equilateral triangles of side length 1. These 10 cardboards are to be put together to form a polygon. If overlapping of cardboards is not allowed, what is the least number of sides of the resulting polygon? (5 marks)

11. 六人參加一次聚會，期間有些人曾經互相握手，而每兩個人最多只會互相握手一次。聚會後，各人記下了自己握手的次數，結果分別是 2、2、3、5、5、n。求 n。（5 分）

Six people joined a gathering. During the gathering, some people shook hands with each other. Any two people have shaken hands with each other at most once. After the gathering, each person recorded the number of times he/she had shaken hands, and the results were 2, 2, 3, 5, 5, n. Find n. (5 marks)

12. 小文寫下了 12 個大於 100 的連續正整數，並發現當中有 n 個質數。求 n 的最大可能值。（5 分）

Lawrence wrote down 12 consecutive positive integers greater than 100 and found that n of them are prime. Find the greatest possible value of n. (5 marks)

13. 在某書店，鉛筆的售價如下：
In a bookstore, the price of pencils is as follows:

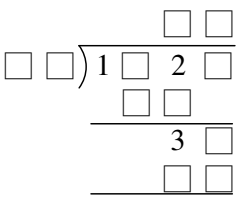
數量 Quantity	每支售價 Unit price
1 – 10	\$0.9
11 – 30	\$0.8
31 – 100	\$0.7
101 – 1000	\$0.6
> 1000	\$0.5

小芬打算買 10 支鉛筆。可是她發現原來買 11 支鉛筆竟然比 10 支便宜。因此，她稱 10 爲「壞數」。類似地，如果某人需要 n 支鉛筆時，可以多買若干支來減低付出的金錢，則稱 n 爲「壞數」。有多少個正整數是「壞數」？（6 分）

Fanny plans to buy 10 pencils. However, she finds that it costs less to buy 11 pencils than 10 pencils. Hence she says that 10 is a 'bad' number. Similarly, n is said to be 'bad' if one can save money by buying extra pencils when one actually needs n pencils. How many positive integers are 'bad'? (6 marks)

14. 圖中顯示一條除式，但當中有些數字留空了。求商（即最頂一行的兩位數）的所有可能值之和。（6 分）

The figure shows a division, but some digits are left out. Find the sum of all possible values of the quotient (i.e. the two-digit number at the top row). (6 marks)



15. 在培正數學邀請賽初賽中，每題的佔分都是 2 至 10 之間的整數（包括 2 和 10），滿分爲 100。某校派出了 n 名學生參加這比賽，而他們各人的得分的數字之和互不相同。求 n 的最大可能值。（6 分）

In the Heat Events of the Pui Ching Invitational Mathematics Competition, the score carried by each question is an integer between 2 and 10 (inclusive), and the full score is 100. A school sends n students for the competition, and it turns out that the sums of digits of the scores of the students are pairwise distinct. Find the greatest possible value of n. (6 marks)

16. 小明和小芬都會按每天的日期中的「日」決定當天穿甚麼顏色的衣服。小明會把日期中的「日」除以 8，如果餘數是偶數便穿紅色衣服，否則穿黃色衣服。小芬則會把日期中的「日」除以 6，如果餘數是 0 或 3 便穿黃色衣服，餘數是 1 或 4 便穿綠色衣服，餘數是 2 或 5 便穿藍色衣服。例如：因爲 27 除以 8 和 6 時餘數都是 3，所以在 1 月 27 日小明和小芬都會穿黃色衣服。在 2007 年中，小明和小芬有多少天穿上不同顏色的衣服？（6 分）

Mike and Fanny both determine the colour of the clothes to wear according to the 'day' of the date. Mike will divide the 'day' by 8 and wear red clothes if the remainder is even, and wear yellow clothes if otherwise. Fanny will divide the 'day' by 6 and wear yellow clothes if the remainder is 0 or 3, green clothes if the remainder is 1 or 4, and blue clothes if the remainder is 2 or 5. For example, since 27 leaves a remainder of 3 when divided by both 8 and 6, both Mike and Fanny wear yellow clothes on 27th January. For how many days in the year 2007 will Mike and Fanny wear clothes of different colours? (6 marks)

17. 2006 位學生圍成一個圈坐著，之後老師派給每人一張黃色、紅色或藍色的咭片。每人都只能看見自己和身旁兩人的咭片。接著，老師問他們看見的咭片的顏色時，每人都回答他們看見黃色、紅色和藍色的咭片各一張。他們當中最少有多少人說了謊？（6 分）

2006 students sit in a circle. The teacher then gives each of them a card which must be yellow, red or blue. Every student can only see his own card and the cards of his two neighbours. When the teacher asks them about the colours of the cards they can see, all students reply that they see a yellow card, a red card and a blue card. At least how many students have lied? (6 marks)

18. 一間薄餅店提供外賣速遞服務。假設每名顧客購買的食品總值都不超過 1000 元，而且都是 0.1 元的倍數，那麼送貨員只要帶備 m 張紙幣和 n 個硬幣，便肯定能夠準確地找贖。（可用的紙幣和硬幣則只有 1000 元、500 元、100 元、50 元、20 元、10 元、5 元、2 元、1 元、0.5 元、0.2 元和 0.1 元共 12 種。）求 $m+n$ 的最小可能值。 （7 分）

A pizza café offers delivery service. Assuming that the total value of the food ordered by each customer does not exceed $1000 and must be a multiple of $0.1, then the delivery staff can ensure accurate change to be given by bringing along m notes and n coins. (There are 12 types of notes and coins, of denominations $1000, $500, $100, $50, $20, $10, $5, $2, $1, $0.5, $0.2 and $0.1). Find the smallest possible value of $m+n$. (7 marks)

19. 一部郵票售賣機售賣面值 1.4 元、1.8 元、2.4 元和 3 元的郵票。若小明需要剛好 12 元的郵票，他所買的郵票可以有多少種不同的面值組合？ （7 分）

A vending machine sells stamps of the values $1.4, $1.8, $2.4 and $3. If Mike needs stamps worth exactly $12, how many different combinations of values are possible for the stamps he buys? (7 marks)

20. 某城市制訂了一條法例，有效期爲 2006 年 5 月 1 日至 2009 年 5 月 1 日（包括首尾兩天）。法例規定未滿 16 歲的人士一律不得進入遊戲機中心，而所有遊戲機中心必須在入口處展示「以下日期以後出生的人士不得進入」的告示，告示的下方則以「年年年年／月月／日日」的八位數字形式展示日期。例如：在 2007 年 1 月 27 日，所展示的日期爲「19910127」。每個數字都必須以一張咭紙製成。假設表示「6」和「9」的數字咭可以互相對調使用，那麼每所遊戲機中心最少要製作多少張數字咭才可確保法例生效期間的每天都能展示出所需的日期？ （7 分）

A city has enacted a law which is effective from 1st May 2006 to 1st May 2009 (inclusive). The law prohibits people aged under 16 to enter game centres. All game centres are required to show at their entrance a sign stating ‘People born after the following date are not allowed to enter’, followed by a 8-digit date represented by the ‘YYYY/MM/DD’ format. For instance, on 27th January 2007, the date shown will be ‘19910127’. Each digit must be shown by a cardboard. Assuming that cardboards for ‘6’ and ‘9’ can be used interchangeably, what is the least number of cardboards that each game centre must make to ensure that the required date can be correctly shown throughout the period of the law? (7 marks)

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