決賽（中四組）

Final Event (Secondary 4)

時限：**2** 小時

Time allowed: 2 hours

參賽者須知：

Instructions to Contestants:

1. 本卷共設甲、乙兩部分，總分爲 100 分。

    This paper is divided into Section A and Section B. The total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

    Unless otherwise stated, all numbers in this paper are in decimal system.

3. 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

    Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

4. 把所有答案塡在答題紙指定的空位上。毋須呈交計算步驟。

    Put your answers on the space provided on the answer sheet. You are not required to hand in your steps of working.

5. 不得使用計算機。

    The use of calculators is not allowed.

6. 本卷的附圖不一定依比例繪成。

    The diagrams in this paper are not necessarily drawn to scale.

甲部（**75分**）
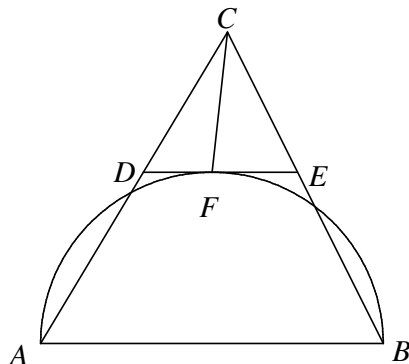
Section A (75 marks)

1. 一個數列的通項是 $15n-50-n^2$。數列中有多少項是正數？　　　　　　（3分）

   A sequence has general term $15n-50-n^2$. How many terms of the sequence are
   positive?　　　　　　　　　　　　　　　　　　　　　　　　　　　(3 marks)

2. 已知 n 是正整數，而 n^2 是 2007 的倍數。求 n 的最小可能值。　　　（3分）

   Given n is a positive integer for which n^2 is a multiple of 2007. Find the smallest
   possible value of n.　　　　　　　　　　　　　　　　　　　　　　(3 marks)

3. 有多少個四位正整數的其中一個數字是 0，而其餘三個數字相同？　　　（3分）

   How many four-digit positive integers have one digit 0 and the other three digits
   equal?　　　　　　　　　　　　　　　　　　　　　　　　　　　　(3 marks)

4. 某長方形的長和闊（以厘米為單位）均為整數。若它的周界為 k 厘米，面積
   為 $k+1$ 平方厘米，求 k。　　　　　　　　　　　　　　　　　　　（4分）

   The length and width (in cm) of a rectangle are both integers. If its perimeter is k
   cm while its area is $(k+1)\,\mathrm{cm}^2$, find k.　　　　　　　　　　　　(4 marks)

5. 若 $x^{x\log x}=10$ 而 $x>1$，求 $\sqrt{x}^{\sqrt{x}}$ 的值。　　　　　　　　　（4分）

   If $x^{x\log x}=10$ where $x>1$, find the value of $\sqrt{x}^{\sqrt{x}}$.　　　　(4 marks)

6. 求一個邊長為 1 的正八邊形的面積。　　　　　　　　　　　　　　　（4分）

   Find the area of a regular octagon with side length 1.　　　　　　　(4 marks)

7　陳先生忘記了某個電話號碼。他只記得電話號碼有八位數字，首個數字是
   2。之後的七個位都是 2、5 或 7，而且三個數字均有出現。那麼陳先生忘記
   了的電話號碼有多少個不同的可能性？　　　　　　　　　　　　　　（5分）

   Mr Chan has forgotten a telephone number. All he can recall is that the telephone
   number consists of eight digits, begins with 2. Also, each of the remaining seven
   digits is 2, 5 or 7 with each of these digits occurring at least once. How many
   possibilities are there for the telephone number which Mr Chan has forgotten?　　(5 marks)

8. 求方程 $(90-x)^2 + \sin^3 x° = 2007$ 所有實數解之和。 （5分）

   Find the sum of all real solutions to the equation $(90-x)^2 + \sin^3 x° = 2007$. (5 marks)

9. 圖中，AB 是半圓的直徑，DE 平行於 AB 且與半圓相切，F 為切點。AD 與 BE 延長後相交於 C。若 $AB=18$、$DF=3$、$EF=6$，求 CF 的長度。 （5分）

   In the figure, AB is a diameter of the semi-circle. DE is parallel to AB and is tangent to the semi-circle at F. AD and BE are produced to meet at C. If $AB=18$, $DF=3$ and $EF=6$, find the length of CF. (5 marks)

10. 八位小朋友玩遊戲。他們每人先抽出一張分別寫上整數「1」至「8」的牌，其中他們的牌各不相同。然後他們隨機分成四對再比較他們牌上的數字，每對當中數字較大者可淘汰對手出線進入準決賽。四位準決賽選手再隨機分成兩對作相同的比較，產生兩位決賽選手。最後兩位決賽選手再按相同的方式決定冠軍。假設子祺和英敏分別抽得「5」和「7」，求他們在比賽途中相遇的概率。 （6分）

    Eight children played a game. Every child drew a card on which an integer from 1 to 8 was written, and their cards were pairwise distinct. They were randomly grouped into 4 pairs and each pair compared the numbers on their cards. The child who had a bigger number in his pair eliminated his opponent and entered the semi-final. The four semi-finalists were randomly grouped into 2 pairs again. The same comparison was done to decide the two finalists. Finally, the champion was decided among the two finalists in the same way. Assuming that Keith and Larry drew the cards '5' and '7' respectively, find the probability that they met during the game. (6 marks)

11. 設 $S = 1+11+111+1111+\cdots+\underbrace{111\ldots111}_{m \text{ 個 } 1}$，其中 $m>1$。若 S 可被 100000 整除，求 m 的最小可能值。 （6分）

    Let $S = 1+11+111+1111+\cdots+\underbrace{111\ldots111}_{m \text{ copies of } 1}$, where $m>1$. If S is divisible by 100000, find the smallest possible value of m. (6 marks)

12. 一次魔術表演共有 k 名觀眾。魔術師要求每位觀眾寫下一個正整數，他並肯定可以找到四名觀眾（他們的數記作 a、b、c 和 d），使得 $(a-b)(c-d)$ 可被 1000 整除。求 k 的最小可能值。　　　　　　　　　　　　　　　　　　　（6分）

In a magic performance show there are k spectators. The magician asks each spectator to write down a positive integer. He is sure that he can find four spectators (whose numbers are denoted by a, b, c and d) such that $(a-b)(c-d)$ is divisible by 1000. Find the smallest possible value of k. (6 marks)

13. 圖中，ABC 是銳角三角形。$\angle ABC$ 和 $\angle ACB$ 的內角平分線交於 D。過 D 作一條平行於 BC 的直線，分別與 AB 和 AC 相交於 E 和 F。若 $AB=15$、$AC=24$，而且 ΔABC 的外接圓面積是 147π，求 EF 的長度。　（7分）
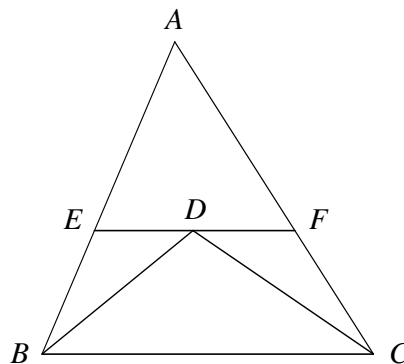
In the figure, ΔABC is acute-angled. The internal bisectors of $\angle ABC$ and $\angle ACB$ meet at D. Through D a straight line parallel to BC is constructed, which meets AB and AC at E and F respectively. If $AB=15$, $AC=24$ and the circumcircle of ΔABC has area 147π, find the length of EF. (7 marks)

14. 有多少種方法把 10 枚相同的棋子放進一個 5×5 的棋盤的其中 10 格，使得每橫行和每直行均有剛好兩枚棋子？　　　　　　（7分）
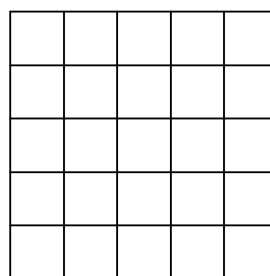
In how many ways can we put 10 identical pieces of chess onto 10 cells of a 5×5 chessboard so that there are exactly 2 chess pieces in each row and in each column? (7 marks)

15. 已知 n 和 $\dfrac{n^3-29\times30\times31\times32}{n+1}$ 皆是整數。求 n 所有可能值之和。　（7分）

Given that both n and $\dfrac{n^3-29\times30\times31\times32}{n+1}$ are integers, find the sum of all possible values of n. (7 marks)

乙部（**25 分**）
Section B (25 marks)

細閱以下資料，然後回答第 16 至第 20 題。
Study the following information and answer Questions 16 to 20.

　　一家科技公司舉辦了一個爲期五天的計算機展覽，展出該公司的各款計算機。所有計算機的屏幕上都只會顯示小數點和 0 至 9 的數字，而且都使用同一款字體：

　　爲吸引更多人入場，大會舉辦了一個「我最喜愛的計算機」選舉。每位入場人士均會獲發一張選票，在展出的計算機中選擇一個他們最喜愛的型號。各人交回選票時可獲贈一張「刮刮咭」和一張 50 元優惠券供在場內購物之用。每張「刮刮咭」上有五格，參加者需刮去其中三格，如果這三格的圖案相同便會中獎。而每張優惠券上的條款如下：

$50 優惠券

使用細則：
1. 憑券購物可作 50 元使用。
2. 每次購物最多只可使用優惠券兩張。
3. 以優惠券購物時不能享用特價優惠。
4. 如購物總額低於優惠券面額，餘額不獲發還。

　　所有入場人士均需在展覽開始前預先登記，並於入場時佩戴該公司的一款「計算機襟章」，同時在襟章的屏幕上打出他們的登記編號以茲識別。每個登記編號都是一個四位正整數，而且各人的登記編號互不相同。

　　在展覽的第一天，大會發現有些入場人士佩戴計算機襟章時上下倒轉了，因而使襟章上顯示出另一個有效的登記編號，例如：「6681」變成了「1899」。大會於是把這些上下倒轉後屏幕顯示成另一個有效登記編號的四位正整數稱爲「壞數」（例如：6681 和 1899 都是「壞數」，1234 則不是「壞數」），並在第二天起重新發出登記編號，使得所有登記編號都不是「壞數」，而新的登記編號依然符合原先的條件。另一方面，在屏幕上下倒轉後仍顯示原數的四位正整數稱爲「好數」，8888 就是其中一個例子。

　　陳先生在展覽的第一天帶同兩名兒子志豪和志強出席，三人就他們的登記編號有一些有趣的發現。以下是他們之間的對話：

　　陳先生說：「我的登記編號是個『好數』呢。」

　　志豪說：「我的登記編號是個『壞數』呢。」

　　志強說：「我的登記編號比爸爸的大 300，比志豪的小 200，既非『好數』亦非『壞數』。」

A technology company had organised a 5-day calculator exhibition displaying the various models of calculators of the company. The screens of all calculators display only the decimal point and digits from 0 to 9 with the same font:

To attract more people, the organiser had included a poll entitled 'My Favourite Calculator'. Each participant was given a ballot paper so that they could select their favourite model among all calculators displayed. When returning the ballot paper, each participant would be given a 'scratch card' and a $50 coupon for purchase in the exhibition. Each 'scratch card' consists of five cells; the player scratches three of them and wins a prize if the pictures in the three cells are the same. The terms and conditions of each $50 coupon are as follows:

$50 COUPON

Terms and Conditions:
1. This coupon may be used as $50 for purchase.
2. At most two coupons may be used for each purchase.
3. No discount will be offered when making purchases with this coupon.
4. No return will be given for purchases less than the face value of the coupon.

All participants were required to register before the start of the exhibition, and wear a 'calculator badge' produced by the company. They must also show their registration number on the screen of the calculator badge for identification purpose. Each registration number is a 4-digit positive integer, and the participants got pairwise different registration numbers.

During the first day of the exhibition, the organiser found that some participants wore the calculator badge upside down, and as a result a different but legal registration number was shown, e.g. '6681' became '1899'. The organiser therefore called those 4-digit positive integers which became another legal registration number on screen when read upside down 'bad' numbers. (For instance, 6681 and 1899 are both 'bad' while 1234 is not.) On the second day, the organiser issued a different set of registration numbers so that all of them were not 'bad' and still satisfied the original conditions. On the other hand, 4-digit integers which read the same when the screen is put upside down are said to be 'good', and 8888 is one such example.

On the first day of the exhibition, Mr Chan participated with his two sons, Henry and Ken. They had some interesting discoveries about their registration numbers, and their conversation was as follows:

"My registration number is a 'good' number," said Mr Chan.

"My registration number is a 'bad' number," said Henry.

"My registration number is 300 greater than Dad's and 200 smaller than Henry's, and is neither 'good' nor 'bad'," said Ken.

16. 求志豪在第一天的登記編號的最大可能值。 （5分）

Find the greatest possible value of Henry's registration number on the first day. (5 marks)

17. 場內展出的其中一款計算機不設「先乘除後加減」的功能，只會按輸入次序作出運算。志豪不知道這事，並使用這款計算機來計算一道形如 $a+b\times c$ 的算式（其中 a、b、c 都是正整數），卻仍然得到正確的答案。求 c。 （3分）

A model of calculator shown in the exhibition does not support the precedence of multiplication and division over addition and subtraction, and only performs calculations based on the order of input. Henry did not know about this and used this model of calculator to compute an expression of the form $a+b\times c$ (where a, b, c are positive integers). Nevertheless, he still obtained the correct answer. Find c. (3 marks)

18. 場內展出的其中一款計算機只有三個鍵：「AC」（可把屏幕上的數重設為 0）、「×3」（可把屏幕上的數乘以 3）和「+2」（可把屏幕上的數加 2）。在按下「AC」鍵後，最少要按鍵多少次才可使屏幕上顯示「5000」？ （7分）

A model of calculator shown in the exhibition consists of only three keys: 'AC' (which resets the number on screen to 0), '×3' (which multiplies 3 to the number on screen) and '+2' (which adds 2 to the number on screen). After pressing 'AC', what is the minimum number of times of pressing keys in order that the screen shows '5000'? (7 marks)

19. 陳先生對場內一款可以顯示 5000 個位的計算機很感興趣。他吩咐志豪和志強利用這款計算機計算一下：若把 $\dfrac{1003}{2007}$ 化成小數，小數點後第 3、第 17 和第 2007 個位之和是甚麼。志豪計出了正確答案，可是志強卻把 $\dfrac{1003}{2007}$ 錯聽成 $\dfrac{1004}{2007}$。那麼，二人所計出的答案之和是多少？ （5分）

Mr Chan was very interested in a model of calculator shown in the exhibition which can display up to 5000 digits on screen. He asked Henry and Ken to use this model of calculator to find the sum of the 3rd, 17th and 2007th digits after the decimal point if $\dfrac{1003}{2007}$ is written as a decimal. Henry got the correct answer while Ken wrongly heard $\dfrac{1003}{2007}$ as $\dfrac{1004}{2007}$. What is the sum of Henry's answer and Ken's answer? (5 marks)

20. 場內展出的其中一款計算機有六個特別鍵，它們的作用如下：

A model of calculator shown in the exhibition contains 6 special keys, and their functions are as follows:

特別鍵 Key	功能 Function
cos	把屏幕上的數 x 變成 $\cos x°$ Changes the number x on screen to $\cos x°$
x^2	把屏幕上的數變成它的平方 Shows the square of the number on screen
1/x	把屏幕上的數變成它的倒數 Shows the reciprocal of the number on screen
−1	把屏幕上的數減 1 Subtracts 1 from the number on screen
$\sqrt{}$	把屏幕上的非負數變成它的非負平方根／對負數沒有反應 Shows the non-negative square root of the non-negative number on screen, and does not respond for a negative number
$\tan^{-1}$	若屏幕上的數為 x，則顯示滿足 $\tan y° = x$ 及 $-90 < y < 90$ 的 y 值 If x is on screen, returns y which satisfies $\tan y° = x$ and $-90 < y < 90$

現時屏幕上顯示 2007。若依上述次序按下六個特別鍵，則屏幕上顯示的數是甚麼？ （5分）

Now the screen shows 2007. If the 6 special keys are pressed in the above order, what will be the number shown on screen? (5 marks)

全卷完

END OF PAPER