第六屆培正數學邀請賽
6th Pui Ching Invitational Mathematics Competition

決賽（中二組）
Final Event (Secondary 2)

時限：2 小時
Time allowed: 2 hours

參賽者須知：
Instructions to Contestants:

1. 本卷共設甲、乙兩部分，總分爲 100 分。
   This paper is divided into Section A and Section B. The total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。
   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。
   Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

4. 把所有答案填在答題紙指定的空位上。毋須呈交計算步驟。
   Put your answers on the space provided on the answer sheet. You are not required to hand in your steps of working.

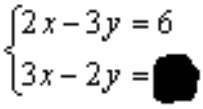
5. 不得使用計算機。
   The use of calculators is not allowed.

6. 本卷的附圖不一定依比例繪成。
   The diagrams in this paper are not necessarily drawn to scale.

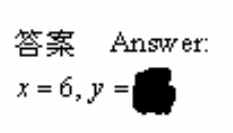
甲部（75 分）

Section A (75 marks)

1. 小婷做聯立方程的練習時，發現其中一道題中第二道方程右方的數字被塗污了（見上圖）。當她從書後翻查這題的答案時，又發現答案中 y 的值被塗污了（見下圖）。已知聯立方程中第二道方程的右方是一個整數，則這個整數是甚麼？



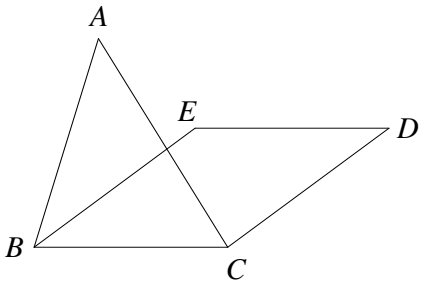
（3 分）



When working on exercise in simultaneous equations, Elaine found that the right hand side of the second equation in a question had been defaced (see the upper figure). When she checked the answer to the question at the end of the book, she again found that the answer for y had been defaced (see the lower figure). Given that the right hand side of the second of the simultaneous equations is an integer, what is this integer?

(3 marks)

2. 圖中，ABC 是三角形、$BCDE$ 是平行四邊形，它們的面積都是 60。求 ΔADE 的面積。



（3 分）

In the figure, ABC is a triangle while $BCDE$ is a parallelogram. They both have area 60. Find the area of ΔADE.

(3 marks)

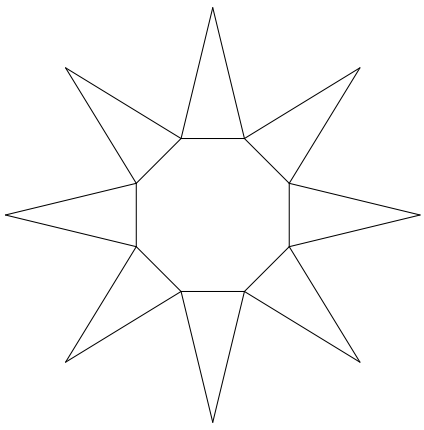
3. 有多少個三位正整數有剛好一個位是「0」？

（3 分）

How many three-digit positive integers contain exactly one digit '0'?

(3 marks)

4. 圖中的摺紙圖樣由一個邊長爲 1 的正八邊形和八個底爲 1、高爲 h 的等腰三角形組成。若圖樣不能摺成一個八角錐體，求 h 的最大可能值。



（4 分）

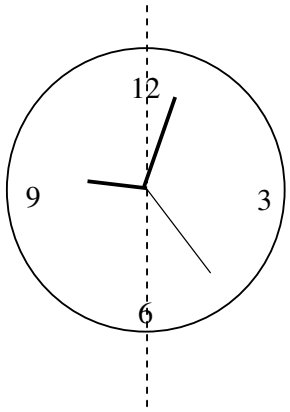
The net in the figure is composed of a regular octagon with side length 1 and eight isosceles triangles with base 1 and height h. If the net cannot be folded into an octagonal pyramid, find the maximum possible value of h.

(4 marks)

5. 某長方形的長和闊（以厘米爲單位）均爲整數。若它的周界爲 k 厘米，面積爲 $k+1$ 平方厘米，求 k。（4分）

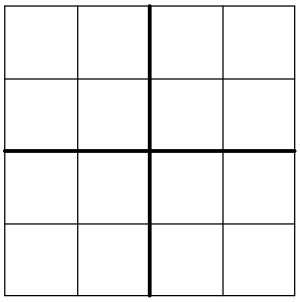
The length and width (in cm) of a rectangle are both integers. If its perimeter is k cm while its area is $(k+1)\,\text{cm}^2$, find k. (4 marks)

6. 某時鐘上有時針、分針和秒針。鐘面上穿過「6」和「12」的刻度的直線（即圖中的虛線）左邊的區域被塗上紅色，右邊的區域則被塗上藍色。一天裏有幾分之幾的時間三支指針都停留在紅色的區域內？（4分）



There are an hour hand, a minute hand and a second hand on a clock. On the face of the clock, the region on the left of the straight line passing through the graduations '6' and '12' (i.e. the dotted line in the figure) is painted in red, while the region on the right is painted in blue. What fraction of the time each day are all three hands inside the red region? (4 marks)

7. 圖中的 4×4 方格表被兩條粗線分成四個 2×2 小方格表。若要在每格填上 1、2、3 或 4，使得每橫行、每直行和這四個 2×2 小方格表中均出現 1、2、3、4 各一次，則填數字的方法共有多少種？（5分）



The figure shows a 4×4 grid, which is divided into four 2×2 small grids by two thick lines. If each cell is to be filled with 1, 2, 3 or 4 in a way such that each row, each column and each of the four 2×2 small grids contains each of 1, 2, 3, 4 once, how many different ways of filling the integers are there? (5 marks)

8. 陳先生忘記了某個電話號碼。他只記得電話號碼有八位數字，首個數字是 2。之後的七個位都是 2、5 或 7，而且三個數字均有出現。那麼陳先生忘記了的電話號碼有多少個不同的可能性？（5分）

Mr Chan has forgotten a telephone number. All he can recall is that the telephone number consists of eight digits, begins with 2. Also, each of the remaining seven digits is 2, 5 or 7 with each of these digits occurring at least once. How many possibilities are there for the telephone number which Mr Chan has forgotten? (5 marks)

9. 求 $\frac{1}{4\times1^4+1}+\frac{2}{4\times2^4+1}+\frac{3}{4\times3^4+1}+\cdots+\frac{100}{4\times100^4+1}$ 的值。（5 分）

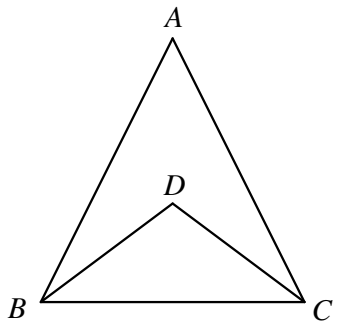
Find the value of $\frac{1}{4\times1^4+1}+\frac{2}{4\times2^4+1}+\frac{3}{4\times3^4+1}+\cdots+\frac{100}{4\times100^4+1}$. (5 marks)

10. 某城市的居民分別住在光明村和仁愛村。若把光明村的村民分成 2007 人一組，則有 m 人剩下來；若把仁愛村的村民分成 2007 人一組，則有 n 人剩下來；若把全市的居民分成 441 人一組，則有 144 人剩下來；若把全市的居民分成 446 人一組，則有 68 人剩下來。求 $m+n$ 的最大可能值。（6 分）

In a city, the citizens reside in the Candle Light Village and the Friendship Village. If the residents of Candle Light Village form groups of 2007, m people will be left ungrouped. If the residents of Friendship Village form groups of 2007, n people will be left ungrouped. If all residents of the city together form groups of 441, 144 people will be left ungrouped. If all residents of the city together form groups of 446, 68 people will be left ungrouped. Find the greatest possible value of $m+n$. (6 marks)

11. 在 ΔABC 中，$AB=AC=5$、$BC=6$。$\angle B$ 和 $\angle C$ 的內角平分線相交於 D。求 AD 的長度。（6 分）

In ΔABC, $AB=AC=5$ and $BC=6$. The internal bisectors of $\angle B$ and $\angle C$ meet at D. Find the length of AD. (6 marks)



12. 有多少對正整數 (x, y) 滿足 $x\le200$ 和 $y\le200$，而且 x、y 和 200 的最大公因數是 1？（6 分）

How many pairs (x, y) of positive integers are there such that $x\le200$, $y\le200$ and that the H.C.F. of x, y and 200 is 1? (6 marks)

13. 已知 n 和 $\frac{n^3-29\times30\times31\times32}{n+1}$ 皆是整數。求 n 所有可能值之和。（7 分）

Given that both n and $\frac{n^3-29\times30\times31\times32}{n+1}$ are integers, find the sum of all possible values of n. (7 marks)

14. 在一個派對裏有三位女士小梅、小蘭、小菊和三位男士小松、小柏和小杉。他們是三對夫婦（不一定按上述的順序）。以下是關於他們的年齡的對話：

小梅說：「小杉比我大。」
小蘭說：「小柏比我大 8 歲。」
小菊說：「這裏六人當中，只有我的丈夫的年齡是奇數。」
小松說：「我們六人的年齡之和是 185。」
小柏說：「三對夫婦裏，妻子的年齡和丈夫的年齡的比例都相同。」
小杉說：「我比妻子大 11 歲。」

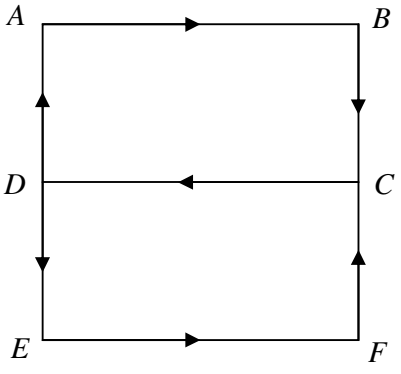
已知他們的年齡都是整數。小松多少歲？ （7 分）

In a party there are 3 ladies, Ada, Betty and Cindy, and 3 men, Jack, Ken and Larry. They are 3 couples (not necessarily in the above order). Here is some conservation about their ages.

Ada said, 'Larry is older than me.'
Betty said, 'Ken is 8 years older than me.'
Cindy said, 'Among all six people here, only my husband has an odd age.'
Jack said, 'The sum of our six ages is 185.'
Ken said, 'The ratios of the age of the wife to that of the husband of all three couples are the same.'
Larry said, 'I am 11 years older than my wife.'

It is known that all their ages are integers. How old is Jack? (7 marks)

15. 如圖所示，*ABFE* 是一個正方形，它的邊長是 2 公里。*C* 和 *D* 分別是 *BF* 和 *AE* 的中點。小華和小文同時在 *C* 點出發，分別沿長方形 *CDAB* 和 *CDEF* 以均速步行，並於 13 小時後首次相遇。如果小華和小文的速度分別是 1 km/h 和 *x* km/h，求 *x* 所有可能值之和。



（7 分）

In the figure, *ABFE* is a square of side length 2 km. *C* and *D* are the mid-points of *BF* and *AE* respectively. Walty and Mathew both start at point *C*, and walk with uniform speeds along rectangles *CDAB* and *CDEF* respectively. They meet for the first time after 13 hours. If the speeds of Walty and Mathew are 1 km/h and *x* km/h respectively, find the sum of all possible values of *x*. (7 marks)

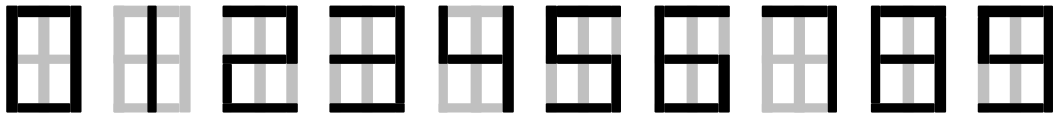
乙部（**25** 分）

Section B (25 marks)

細閱以下資料，然後回答第 16 至第 20 題。

Study the following information and answer Questions 16 to 20.

一家科技公司舉辦了一個爲期五天的計算機展覽，展出該公司的各款計算機。所有計算機的屏幕上都只會顯示小數點和 0 至 9 的數字，而且都使用同一款字體：



爲吸引更多人入場，大會舉辦了一個「我最喜愛的計算機」選舉。每位入場人士均會獲發一張選票，在展出的計算機中選擇一個他們最喜愛的型號。各人交回選票時可獲贈一張「刮刮咭」和一張 50 元優惠券供在場內購物之用。每張「刮刮咭」上有五格，參加者需刮去其中三格，如果這三格的圖案相同便會中獎。而每張優惠券上的條款如下：

$50 優惠券

使用細則：

1. 憑券購物可作 50 元使用。
2. 每次購物最多只可使用優惠券兩張。
3. 以優惠券購物時不能享用特價優惠。
4. 如購物總額低於優惠券面額，餘額不獲發還。

所有入場人士均需在展覽開始前預先登記，並於入場時佩戴該公司的一款「計算機襟章」，同時在襟章的屏幕上打出他們的登記編號以茲識別。每個登記編號都是一個四位正整數，而且各人的登記編號互不相同。

在展覽的第一天，大會發現有些入場人士佩戴計算機襟章時上下倒轉了，因而使襟章上顯示出另一個有效的登記編號，例如：「6681」變成了「1899」。大會於是把這些上下倒轉後屏幕顯示成另一個有效登記編號的四位正整數稱爲「壞數」（例如：6681 和 1899 都是「壞數」，1234 則不是「壞數」），並在第二天起重新發出登記編號，使得所有登記編號都不是「壞數」，而新的登記編號依然符合原先的條件。另一方面，在屏幕上下倒轉後仍顯示原數的四位正整數稱爲「好數」，8888 就是其中一個例子。

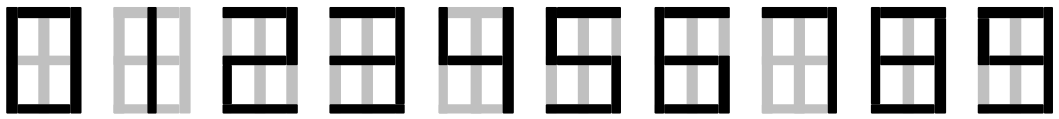
陳先生在展覽的第一天帶同兩名兒子志豪和志強出席，三人就他們的登記編號有一些有趣的發現。以下是他們之間的對話：

陳先生說：「我的登記編號是個『好數』呢。」

志豪說：「我的登記編號是個『壞數』呢。」

志強說：「我的登記編號比爸爸的大 300，比志豪的小 200，既非『好數』亦非『壞數』。」

A technology company had organised a 5-day calculator exhibition displaying the various models of calculators of the company. The screens of all calculators display only the decimal point and digits from 0 to 9 with the same font:



To attract more people, the organiser had included a poll entitled 'My Favourite Calculator'. Each participant was given a ballot paper so that they could select their favourite model among all calculators displayed. When returning the ballot paper, each participant would be given a 'scratch card' and a $50 coupon for purchase in the exhibition. Each 'scratch card' consists of five cells; the player scratches three of them and wins a prize if the pictures in the three cells are the same. The terms and conditions of each $50 coupon are as follows:

$50 COUPON

Terms and Conditions:

1. This coupon may be used as $50 for purchase.
2. At most two coupons may be used for each purchase.
3. No discount will be offered when making purchases with this coupon.
4. No return will be given for purchases less than the face value of the coupon.

All participants were required to register before the start of the exhibition, and wear a 'calculator badge' produced by the company. They must also show their registration number on the screen of the calculator badge for identification purpose. Each registration number is a 4-digit positive integer, and the participants got pairwise different registration numbers.

During the first day of the exhibition, the organiser found that some participants wore the calculator badge upside down, and as a result a different but legal registration number was shown, e.g. '6681' became '1899'. The organiser therefore called those 4-digit positive integers which became another legal registration number on screen when read upside down 'bad' numbers. (For instance, 6681 and 1899 are both 'bad' while 1234 is not.) On the second day, the organiser issued a different set of registration numbers so that all of them were not 'bad' and still satisfied the original conditions. On the other hand, 4-digit integers which read the same when the screen is put upside down are said to be 'good', and 8888 is one such example.

On the first day of the exhibition, Mr Chan participated with his two sons, Henry and Ken. They had some interesting discoveries about their registration numbers, and their conversation was as follows:

"My registration number is a 'good' number," said Mr Chan.

"My registration number is a 'bad' number," said Henry.

"My registration number is 300 greater than Dad's and 200 smaller than Henry's, and is neither 'good' nor 'bad'," said Ken.

16. 假設登記入場人數爲 4000 人。在五天展覽中，入場人數分別有 3200、3600、2800、3000 和 3400 人。若其中 n 人在五天均有入場，而每人每天只可入場一次，求 n 的最小可能值。（4 分）

Assume that 4000 people had registered for admission, and that in the 5 days of exhibition, the admission figures were 3200, 3600, 2800, 3000 and 3400 respectively. If n people had participated on all 5 days, and each participant might only enter the exhibition once each day, find the smallest possible value of n. (4 marks)

17. 在「我最喜愛的計算機」選舉結束後，大會公佈了每款計算機的得票百分率，全部均準確至小數點後一位（例如：5.8%、12.0% 等）。志豪發現，其中兩款計算機的所得票數並不相同，可是得票百分率卻相同。求選票總數的最小可能值。（5 分）

After the 'My Favourite Calculator' poll, the organiser announced the percentage of votes that each calculator got correct to 1 decimal place (e.g. 5.8%, 12.0% etc.). Henry found that two calculators got different numbers of votes but turned out to have the same percentage. Find the smallest possible value of the total number of votes. (5 marks)

18. 陳先生集齊了自己和兩名兒子共三張 50 元優惠券，打算購買 10 隻「計算機手錶」。這些「計算機手錶」每隻原價 32 元，但有特價八折優惠。那麼陳先生最少要付款多少元？（5 分）

Mr Chan collected the three $50 coupons from himself and his two sons, and planned to buy 10 'calculator watches'. Each 'calculator watch' is marked for sale at 32 dollars, but a 20% discount is available. What is the minimum amount (in dollars) that Mr Chan had to pay? (5 marks)

19. 求志豪在第一天的登記編號的最大可能值。（5 分）

Find the greatest possible value of Henry's registration number on the first day. (5 marks)

20. 場內展出的其中一款計算機只能處理整數。當運算結果不是整數時，計算機會把小數部分截去。例如：計算 $25\div6\times9$ 時，因爲 $25\div6$ 會變成 4，因此最後答案會是 36。志強使用這款計算機來計算 $1000\div n\times n$，其中 n 是小於 1000 的正整數。若志強所得的答案是 980，n 有多少個不同的可能值？（6 分）

A model of calculator shown in the exhibition can only handle integers. If the result of a computation is not an integer, the decimal part will be truncated. For instance, $25\div6\times9$ will be computed to be 36 because $25\div6$ becomes 4. Ken used this model of calculator to compute $1000\div n\times n$, where n is a positive integer less than 1000. How many different possible values of n are there if the result Ken obtained was 980? (6 marks)

全卷完

END OF PAPER