第六屆培正數學邀請賽
6th Pui Ching Invitational Mathematics Competition

決賽（中一組）
Final Event (Secondary 1)

時限：2 小時
Time allowed: 2 hours

參賽者須知：
Instructions to Contestants:

1. 本卷共設甲、乙兩部分，總分爲 100 分。
   This paper is divided into Section A and Section B. The total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。
   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。
   Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

4. 把所有答案填在答題紙指定的空位上。毋須呈交計算步驟。
   Put your answers on the space provided on the answer sheet. You are not required to hand in your steps of working.

5. 不得使用計算機。
   The use of calculators is not allowed.

6. 本卷的附圖不一定依比例繪成。
   The diagrams in this paper are not necessarily drawn to scale.

甲部（75 分）

Section A (75 marks)

1. 考慮算式「(−1) □ (−1) □ (−1) □ (−1)」。若在三個空格中各自填上「+」、「−」、「×」或「÷」，則算式的最大可能值是多少？（3 分）

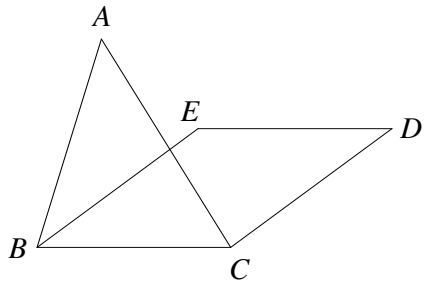
   Consider the expression '(−1) □ (−1) □ (−1) □ (−1)'. If one of the symbols '+', '−', '×' or '÷' is to be filled into each of the three blanks, what is the greatest value of the expression that may be obtained? (3 marks)

2. 某學校有小學部、初中部和高中部。小學部的上課時間表採用 5 天循環週制，分別稱爲上課日 A 至上課日 E（即首五天分別是上課日 A 至上課日 E，第六天又回到上課日 A，如此類推）。初中部的上課時間表採用 7 天循環週制，分別稱爲上課日 F 至上課日 L。高中部的上課時間表則採用 14 天循環週制，分別稱爲上課日 M 至上課日 Z。在學校大門，有一塊牌子寫上當天各部的上課日，例如第一天是「AFM」、第二天是「BGN」，如此類推。若各部的上課日子都相同，那麼牌子上的三個英文字母有多少個不同的組合？（3 分）

   A school comprises a primary school section, a middle school section and a high school section. In the primary school section, the time table runs on a 5-day cycle basis where the schooldays are called Day A to Day E (i.e. the first 5 days are Day A to Day E, the 6th day is back to Day A again and so on). In the middle school section, the time table runs on a 7-day cycle basis where the schooldays are called Day F to Day L. In the high school section, the time table runs on a 14-day cycle basis where the schooldays are called Day M to Day Z. At the entrance of the school there is a board showing the day of school for each section, e.g. 'AFM' for the first day, 'BGN' for the second day and so on. Given that all sections are having school on exactly the same dates, how many different combinations are there for the three letters appearing on the board? (3 marks)

3. 圖中，*ABC* 是三角形、*BCDE* 是平行四邊形，它們的面積都是 60。求 ΔADE 的面積。（3 分）

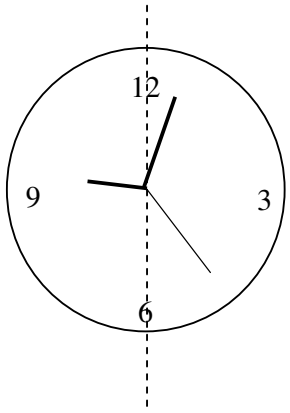
   In the figure, *ABC* is a triangle while *BCDE* is a parallelogram. They both have area 60. Find the area of ΔADE. (3 marks)



4. 某長方形的長和闊（以厘米爲單位）爲不同的整數。若它的周界爲 k 厘米，面積爲 k 平方厘米，求 k。（4 分）

   The length and width (in cm) of a rectangle are distinct integers. If its perimeter is k cm while its area is k cm^2, find k. (4 marks)

5. 某時鐘上有時針、分針和秒針。鐘面上穿過「6」和「12」的刻度的直線（即圖中的虛線）左邊的區域被塗上紅色，右邊的區域則被塗上藍色。一天裏有幾分之幾的時間三支指針都停留在紅色的區域內？



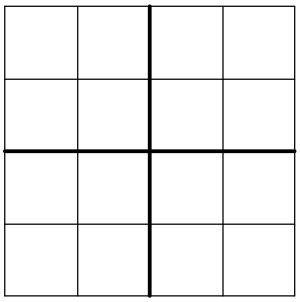
（4 分）

There are an hour hand, a minute hand and a second hand on a clock. On the face of the clock, the region on the left of the straight line passing through the graduations '6' and '12' (i.e. the dotted line in the figure) is painted in red, while the region on the right is painted in blue. What fraction of the time each day are all three hands inside the red region? (4 marks)

6. 香港通用的硬幣有七種，面值分別爲 10 元、5 元、2 元、1 元、5 角、2 角和 1 角。若有每種硬幣各一個，則可以組成多少個不同的正數金額？（4 分）

In Hong Kong, there are seven types of coins, of denominations $10, $5, $2, $1, 50 cents, 20 cents and 10 cents respectively. If one has one coin of each type, how many different positive amounts can be formed? (4 marks)

7. 圖中的 4×4 方格表被兩條粗線分成四個 2×2 小方格表。若要在每格填上 1、2、3 或 4，使得每橫行、每直行和這四個 2×2 小方格表中均出現 1、2、3、4 各一次，則填數字的方法共有多少種？



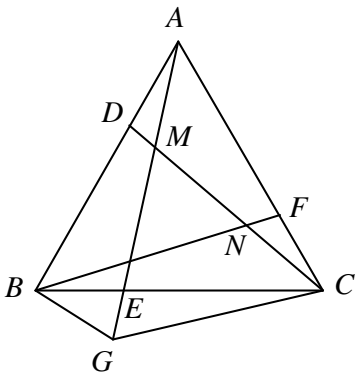
（5 分）

The figure shows a 4×4 grid, which is divided into four 2×2 small grids by two thick lines. If each cell is to be filled with 1, 2, 3 or 4 in a way such that each row, each column and each of the four 2×2 small grids contains each of 1, 2, 3, 4 once, how many different ways of filling the integers are there? (5 marks)

8. 求 $\frac{1}{4\times1^4+1}+\frac{2}{4\times2^4+1}+\frac{3}{4\times3^4+1}+\cdots+\frac{100}{4\times100^4+1}$ 的值。（5 分）

Find the value of $\frac{1}{4\times1^4+1}+\frac{2}{4\times2^4+1}+\frac{3}{4\times3^4+1}+\cdots+\frac{100}{4\times100^4+1}$. (5 marks)

9. 圖中，ABC 是等邊三角形，D、E、F 分別是 AB 、 BC 和 CA 上的點，使得 $AD = BE = CF$ 。G 是 AE 延線上的一點，使得 $\angle BCG = \angle ACD$。CD 與 AE 和 BF 分別交於 M 和 N。若 $DM = 2$ 、 $MN = 30$ 、 $NC = 10$ ，求 EG。 （5 分）



In the figure, ΔABC is equilateral. D, E, F are points on AB, BC and CA respectively such that $AD = BE = CF$. G is a point on AE produced such that $\angle BCG = \angle ACD$. CD meets AE and BF at M and N respectively. If $DM = 2$, $MN = 30$ and $NC = 10$, find EG. (5 marks)

10. 有些四位正整數的四位數字互不相同。若把這些四位數各自除以 100，然後把所有餘數加起來，答案是多少？ （6 分）

Some four-digit positive integers have all four digits distinct. If each of them is divided by 100 and the remainders are added up, what will be the sum? (6 marks)

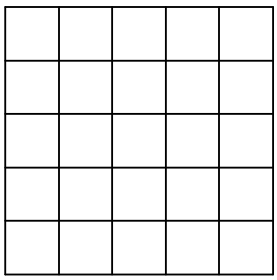
11. 設 $S = 1+11+111+1111+\cdots+\underbrace{111\ldots111}_{m\text{ 個 }1}$，其中 $m > 1$。若 S 可被 100000 整除，求 m 的最小可能值。 （6 分）

Let $S = 1+11+111+1111+\cdots+\underbrace{111\ldots111}_{m\text{ copies of }1}$, where $m > 1$. If S is divisible by 100000, find the smallest possible value of m. (6 marks)

12. 一次魔術表演共有 k 名觀眾。魔術師要求每位觀眾寫下一個正整數，他並肯定可以找到四名觀眾（他們的數記作 a、b、c 和 d），使得 $(a-b)(c-d)$ 可被 1000 整除。求 k 的最小可能值。 （6 分）

In a magic performance show there are k spectators. The magician asks each spectator to write down a positive integer. He is sure that he can find four spectators (whose numbers are denoted by a, b, c and d) such that $(a-b)(c-d)$ is divisible by 1000. Find the smallest possible value of k. (6 marks)

13. 有多少種方法把 10 枚相同的棋子放進一個 5×5 的棋盤的其中 10 格，使得每橫行和每直行均有剛好兩枚棋子？ （7 分）



In how many ways can we put 10 identical pieces of chess onto 10 cells of a 5×5 chessboard so that there are exactly 2 chess pieces in each row and in each column? (7 marks)

14. 在一個派對裏有三位女士小梅、小蘭、小菊和三位男士小松、小柏和小杉。他們是三對夫婦（不一定按上述的順序）。以下是關於他們的年齡的對話：

小梅說：「小杉比我大。」
小蘭說：「小柏比我大 8 歲。」
小菊說：「這裏六人當中，只有我的丈夫的年齡是奇數。」
小松說：「我們六人的年齡之和是 185。」
小柏說：「三對夫婦裏，妻子的年齡和丈夫的年齡的比例都相同。」
小杉說：「我比妻子大 11 歲。」

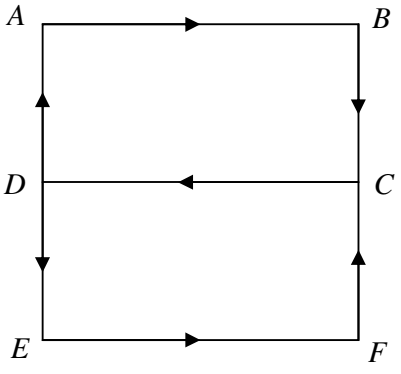
已知他們的年齡都是整數。小松多少歲？ （7 分）

In a party there are 3 ladies, Ada, Betty and Cindy, and 3 men, Jack, Ken and Larry. They are 3 couples (not necessarily in the above order). Here is some conservation about their ages.

Ada said, 'Larry is older than me.'
Betty said, 'Ken is 8 years older than me.'
Cindy said, 'Among all six people here, only my husband has an odd age.'
Jack said, 'The sum of our six ages is 185.'
Ken said, 'The ratios of the age of the wife to that of the husband of all three couples are the same.'
Larry said, 'I am 11 years older than my wife.'

It is known that all their ages are integers. How old is Jack? (7 marks)

15. 如圖所示，*ABFE* 是一個正方形，它的邊長是 2 公里。*C* 和 *D* 分別是 *BF* 和 *AE* 的中點。小華和小文同時在 *C* 點出發，分別沿長方形 *CDAB* 和 *CDEF* 以均速步行，並於 13 小時後首次相遇。如果小華和小文的速度分別是 1 km/h 和 *x* km/h，求 *x* 所有可能值之和。



（7 分）

In the figure, *ABFE* is a square of side length 2 km. *C* and *D* are the mid-points of *BF* and *AE* respectively. Walty and Mathew both start at point *C*, and walk with uniform speeds along rectangles *CDAB* and *CDEF* respectively. They meet for the first time after 13 hours. If the speeds of Walty and Mathew are 1 km/h and *x* km/h respectively, find the sum of all possible values of *x*. (7 marks)

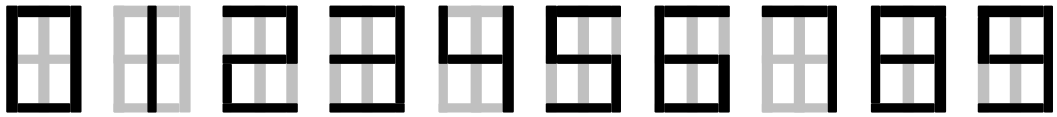
乙部（**25** 分）

Section B (25 marks)

細閱以下資料，然後回答第 16 至第 20 題。

Study the following information and answer Questions 16 to 20.

一家科技公司舉辦了一個爲期五天的計算機展覽，展出該公司的各款計算機。所有計算機的屏幕上都只會顯示小數點和 0 至 9 的數字，而且都使用同一款字體：



爲吸引更多人入場，大會舉辦了一個「我最喜愛的計算機」選舉。每位入場人士均會獲發一張選票，在展出的計算機中選擇一個他們最喜愛的型號。各人交回選票時可獲贈一張「刮刮咭」和一張 50 元優惠券供在場內購物之用。每張「刮刮咭」上有五格，參加者需刮去其中三格，如果這三格的圖案相同便會中獎。而每張優惠券上的條款如下：

$50 優惠券

使用細則：

1. 憑券購物可作 50 元使用。
2. 每次購物最多只可使用優惠券兩張。
3. 以優惠券購物時不能享用特價優惠。
4. 如購物總額低於優惠券面額，餘額不獲發還。

所有入場人士均需在展覽開始前預先登記，並於入場時佩戴該公司的一款「計算機襟章」，同時在襟章的屏幕上打出他們的登記編號以茲識別。每個登記編號都是一個四位正整數，而且各人的登記編號互不相同。

在展覽的第一天，大會發現有些入場人士佩戴計算機襟章時上下倒轉了，因而使襟章上顯示出另一個有效的登記編號，例如：「6681」變成了「1899」。大會於是把這些上下倒轉後屏幕顯示成另一個有效登記編號的四位正整數稱爲「壞數」（例如：6681 和 1899 都是「壞數」，1234 則不是「壞數」），並在第二天起重新發出登記編號，使得所有登記編號都不是「壞數」，而新的登記編號依然符合原先的條件。另一方面，在屏幕上下倒轉後仍顯示原數的四位正整數稱爲「好數」，8888 就是其中一個例子。

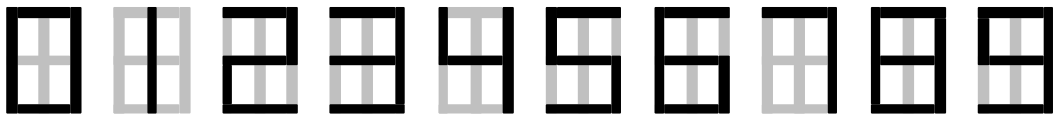
陳先生在展覽的第一天帶同兩名兒子志豪和志強出席，三人就他們的登記編號有一些有趣的發現。以下是他們之間的對話：

陳先生說：「我的登記編號是個『好數』呢。」

志豪說：「我的登記編號是個『壞數』呢。」

志強說：「我的登記編號比爸爸的大 300，比志豪的小 200，既非『好數』亦非『壞數』。」

A technology company had organised a 5-day calculator exhibition displaying the various models of calculators of the company. The screens of all calculators display only the decimal point and digits from 0 to 9 with the same font:



To attract more people, the organiser had included a poll entitled 'My Favourite Calculator'. Each participant was given a ballot paper so that they could select their favourite model among all calculators displayed. When returning the ballot paper, each participant would be given a 'scratch card' and a $50 coupon for purchase in the exhibition. Each 'scratch card' consists of five cells; the player scratches three of them and wins a prize if the pictures in the three cells are the same. The terms and conditions of each $50 coupon are as follows:

$50 COUPON

Terms and Conditions:

1. This coupon may be used as $50 for purchase.
2. At most two coupons may be used for each purchase.
3. No discount will be offered when making purchases with this coupon.
4. No return will be given for purchases less than the face value of the coupon.

All participants were required to register before the start of the exhibition, and wear a 'calculator badge' produced by the company. They must also show their registration number on the screen of the calculator badge for identification purpose. Each registration number is a 4-digit positive integer, and the participants got pairwise different registration numbers.

During the first day of the exhibition, the organiser found that some participants wore the calculator badge upside down, and as a result a different but legal registration number was shown, e.g. '6681' became '1899'. The organiser therefore called those 4-digit positive integers which became another legal registration number on screen when read upside down 'bad' numbers. (For instance, 6681 and 1899 are both 'bad' while 1234 is not.) On the second day, the organiser issued a different set of registration numbers so that all of them were not 'bad' and still satisfied the original conditions. On the other hand, 4-digit integers which read the same when the screen is put upside down are said to be 'good', and 8888 is one such example.

On the first day of the exhibition, Mr Chan participated with his two sons, Henry and Ken. They had some interesting discoveries about their registration numbers, and their conversation was as follows:

"My registration number is a 'good' number," said Mr Chan.

"My registration number is a 'bad' number," said Henry.

"My registration number is 300 greater than Dad's and 200 smaller than Henry's, and is neither 'good' nor 'bad'," said Ken.

16. 試估算第 7 頁英文版的參考資料中（包括「計算機字型」和「優惠券」上的條款）英文字母、數字和標點符號的總數，答案準確至最接近千位。（3 分）

Estimate the total number of English letters, digits and punctuation marks in the English version of the reference materials on page 7 (including the 'calculator fonts' and the terms and conditions of the coupon). Correct your answer to the nearest thousand. (3 marks)

17. 假設登記入場人數爲 4000 人，而五天展覽中平均每天入場人數爲 3300 人。若其中 n 人在五天均有入場，而每人每天只可入場一次，求 n 的最小可能值。（5 分）

Assume that 4000 people had registered for admission, and that in the 5 days of exhibition, the average admission figure per day was 3300. If n people had participated on all 5 days, and each participant might only enter the exhibition once each day, find the smallest possible value of n. (5 marks)

18. 在「我最喜愛的計算機」選舉結束後，大會公佈了每款計算機的得票百分率，全部均準確至小數點後一位（例如：5.8%、12.0% 等）。志豪發現，其中兩款計算機的所得票數並不相同，可是得票百分率卻相同。求選票總數的最小可能值。（5 分）

After the 'My Favourite Calculator' poll, the organiser announced the percentage of votes that each calculator got correct to 1 decimal place (e.g. 5.8%, 12.0% etc.). Henry found that two calculators got different numbers of votes but turned out to have the same percentage. Find the smallest possible value of the total number of votes. (5 marks)

19. 求登記入場人數的最大可能值。（7 分）

Find the greatest possible value of the number of people registered for admission. (7 marks)

20. 場內展出的其中一款計算機只有三個鍵：「AC」（可把屏幕上的數重設爲 0）、「×2」（可把屏幕上的數乘以 2）和「+1」（可把屏幕上的數加 1）。在按下「AC」鍵後，最少要按鍵多少次才可使屏幕上顯示「2007」？（5 分）

A model of calculator shown in the exhibition consists of only three keys: 'AC' (which resets the number on screen to 0), '×2' (which multiplies 2 to the number on screen) and '+1' (which adds 1 to the number on screen). After pressing 'AC', what is the minimum number of times of pressing keys in order that the screen shows '2007'? (5 marks)

全卷完

END OF PAPER