培正數學邀請賽

Pui Ching Invitational Mathematics Competition

初賽（中一組）

Heat Event (Secondary 1)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

模擬試題

Sample Paper

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

   There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

   All answers are integers between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

   The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

   The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 4 題，每題 3 分。

Questions 1 to 4 each carries 3 marks.

1. 100000000001 除以 3 時，餘數是多少？

   What is the remainder when 100000000001 is divided by 3?

2. 求最小的正整數 n，使得 $80-n$ 和 $80+n$ 均爲質數。

   Find the smallest positive integer n for which both $80-n$ and $80+n$ are prime.

3. 求 1122、2233 和 3344 的最大公因數。

   Find the highest common factor (H.C.F.) of 1122, 2233 and 3344.

4. 某長方體的長是闊的兩倍，高是闊的三倍。若它的體積爲 48，它的總表面積是多少？

   A cuboid has length twice its width and height three times its width. If its volume is 48, what is its total surface area?

第 5 至第 8 題，每題 4 分。

Questions 5 to 8 each carries 4 marks.

5. 在某餐廳中，午市套餐包括餐湯、主菜及飲品。顧客可從 3 款餐湯、6 款主菜及 12 款飲品中各選一款。問午市套餐共有多少個不同的組合？

   In a canteen, a set lunch consists of a soup, a main dish and a drink. Customers could choose one among each of the 3 choices of soups, 6 choices of main dishes and 12 choices of drinks. How many different combinations of the set lunch are there?

6. 求 $\underbrace{1+1-1\times1\div1+1-1\times1\div1+\cdots}_{1234\text{個「1」}}$ 的值。

   Find the value of $\underbrace{1+1-1\times1\div1+1-1\times1\div1+\cdots}_{1234\text{'1's}}$.

7. 陳先生參加了一個有獎遊戲。這個有獎遊戲的玩法如下：每位參加者需要回答獎券上的 10 道是非題。如果他能夠答對 5 題或以上則中獎，否則便算落空。陳先生最少需要參加遊戲多少次才可以確保最少有一次中獎？（註：每次獎券上的問題皆相同。）

Mr Chan participates in a game. The rule of the game is as follows: Each player needs to answer 10 'True or False' questions on a ticket. If he can answer at least 5 of them correctly, he wins a prize. Otherwise, he loses. At least how many times must Mr Chan play this game so as to guarantee at least 1 prize? (Note: The same questions are set on every ticket.)

8. 若 n 除以 2004 時的餘數爲 1234，則 $2n$ 除以 2004 時的餘數是多少？

If n leaves a remainder of 1234 when divided by 2004, what is the remainder when $2n$ is divided by 2004?

第 9 至第 12 題，每題 5 分。

Questions 9 to 12 each carries 5 marks.

9. 某正方形的周界爲 k 單位，面積爲 k 平方單位。求 k。

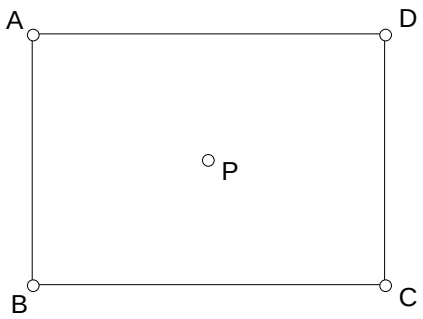
A square has perimeter k units and area k square units. Find k.

10. 某國家只有一元、三元、五元及七元四種硬幣。若不設找贖，共有多少種方法付款恰好十元？

In a country, there are only four types of coins, \$1, \$3, \$5 and \$7. In how many ways can exactly \$10 be paid if no change is allowed?

11. 圖中，P 爲長方形 $ABCD$ 的中心。若 P 到 AB 的距離爲 P 到 BC 的距離之兩倍，且 $ABCD$ 的周界爲 120，求 $ABCD$ 的面積。

In the figure, P is the centre of rectangle $ABCD$. If the distance from P to AB is twice the distance from P to BC, and the perimeter of $ABCD$ is 120, find the area of $ABCD$.



12. 求 $7\times9\times11\times\cdots\times1999\times2001\times2003$ 的個位數字。

Find the unit digit of $7\times9\times11\times\cdots\times1999\times2001\times2003$.

第 13 至第 16 題，每題 6 分。

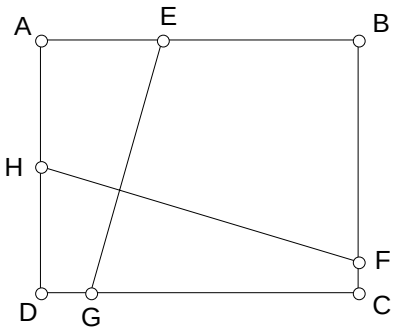
Questions 13 to 16 each carries 6 marks.

13. 一個三角形被放在一個邊長爲 10 的正方形內。三角形的最大可能面積是多少？

A triangle is put inside a square of side length 10. What is the largest possible area of the triangle?

14. 如圖所示，*ABCD* 爲長方形，*E*、*F*、*G*、*H* 分別爲 *AB*、*BC*、*CD*、*DA* 上的點，且 $EG=3$，$FH=4$。求長方形周界的最大可能值。

In the figure, *ABCD* is a rectangle. *E*, *F*, *G*, *H* are points on *AB*, *BC*, *CD*, *DA* respectively with $EG=3$, $FH=4$. Find the largest possible perimeter of the rectangle.



15. 一位老師把 2003 名學生分成若干組，並記下了每組的人數。他最多會記下多少個不同的正整數？

A teacher divides 2003 students into a certain number of groups, and the number of students in each group is recorded. At most how many different positive integers can be recorded?

16. 小明、小強和小霖在猜一輛玩具車的售價。小明猜 123 元，小強猜 129 元，小霖猜 141 元。結果他們的估計中，誤差最少爲 4 元，最多爲 10 元。玩具車的實際售價是多少元？

Alan, Peter and Tom each made a guess of the price of a toy car. Alan guessed \$123, Peter guessed \$129 and Tom guessed \$141. Among their guesses, the smallest error was \$4 and the largest error was \$10. Find, in dollars, the actual price of the toy car.

第 17 至第 20 題，每題 7 分。

Questions 17 to 20 each carries 7 marks.

17. 求最小的質數 p，使得 $2002-p$ 和 $2002+p$ 均爲質數。

Find the smallest prime number p for which both $2002-p$ and $2002+p$ are prime.

18. 陳先生有兩名兒子，他們的年齡相差一歲。在 1997 年，陳先生的年齡是兩名兒子年齡之和的八倍。在 2002 年，陳先生的年齡是兩名兒子年齡之和的三倍。當陳先生的長子出生時，陳先生的年齡是多少？

Mr Chan has two sons whose ages differ by 1. In 1997, the age of Mr Chan was 8 times the sum of his sons'. In 2002, the age of Mr Chan was 3 times the sum of his sons'. How old was Mr Chan when his elder son was born?

19. 某間超級市場只有一台收銀機。現有十名客人在排隊，其中一人選購了 1 件貨品，一人選購了 2 件貨品，如此類推，購物最多的客人選購了 10 件貨品。已知每選購一件貨品付款需時一分鐘（例如：若選購了 5 件貨品，付款過程便需時 5 分鐘）。設 a_1 、a_2 、…、a_{10} 分別代表十名客人的輪候時間（以分鐘爲單位），輪候時間包括繳付自己所購貨品的時間。求 $a_1+a_2+\cdots+a_{10}$ 的最小可能值。

A supermarket has only one cashier counter. Now 10 customers queue up for paying. One of them purchases 1 piece of goods, one purchases 2 pieces of goods, and so on, and one purchases 10 pieces of goods. It is known that the paying process takes 1 minute for each piece of good bought (for instance, the paying process will take 5 minutes if one buys 5 pieces of goods). Let a_1 , a_2, …, a_{10} denote the waiting time (in minutes) for the 10 customers. Waiting time includes the time paying for one's own goods. Find the minimum possible value of $a_1+a_2+\cdots+a_{10}$.

20. 已知 $\frac{1}{13}=0.\dot{0}76923\dot{}$。若把 $\frac{23}{130}$ 以小數表示，小數點後首 2003 位數字之和是多少？

Given $\frac{1}{13}=0.\dot{0}76923\dot{}$. If $\frac{23}{130}$ is written as a decimal, what is the sum of the first 2003 digits after the decimal point?

全卷完

END OF PAPER

答案：

Answers:

1. 2
2. 9
3. 11
4. 88

5. 216
6. 2
7. 2
8. 464

9. 16
10. 9
11. 800
12. 5

13. 50
14. 14
15. 62
16. 133

17. 1999
18. 37
19. 220
20. 9016

註：

1. 初賽試題的難度較決賽爲低。
2. 本屆決賽形式與去屆個人賽相同（唯一改變是時限由 1 小時 30 分增至 2 小時），參賽者可參閱去屆的個人賽試題。

Remarks:

1. Questions in the Heat Event are easier than those in the Final Event.
2. The Final Events this year will be of the same format as the Individual Events of the previous competition. (The only difference is that the time allowed will be lengthened from 1 hour 30 minutes to 2 hours). Contestants may refer to the Indiviudal Events of the previous competition.